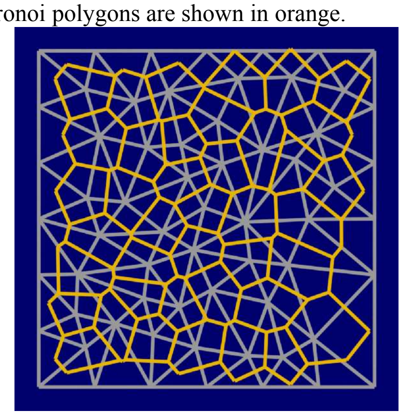
3000+ variations of the Voronoi diagram (sap_0296)

Saty Raghavachary, Mark Matthews
DreamWorks Feature Animation
{saty|mmathews}@anim.dreamworks.com

Introduction

The Voronoi diagram, which describes a way to partition space, is very useful in a variety of fields ranging from astronomy to computer graphics to zoology. This sketch discusses a general idea which makes it possible to easily construct many alternatives to the classical Voronoi polygon. These alternatives could give rise to novel applications in the areas of fracture synthesis, mosaic generation and surface texture warping.

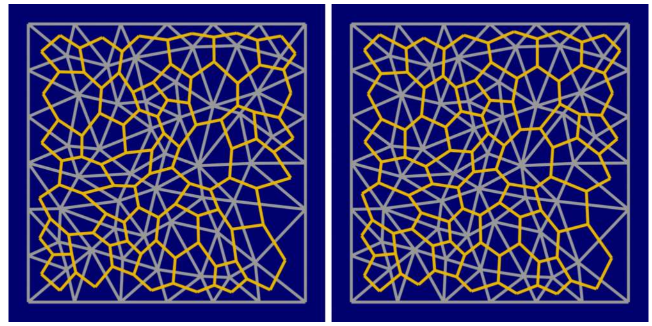
A popular way to construct a Voronoi polygon network is to start with a collection of 'seed' points on a plane (for the 2D case), and triangulate the seed collection using the Delaunay triangulation scheme. The Voronoi diagram, being the mathematical dual of the Delaunay triangulation, is constructed by sequentially finding the circumcenter of each triangle (intersection point of the perpendicular bisectors of the three edges), in the fan of triangles surrounding each vertex in the Delaunay triangle set. These circumcenters are the vertices of the resulting Voronoi polygon, which describes the area influenced by the Delaunay vertex. This is shown in the figure below, where the seed points are the vertices of the Delaunay triangle network (in gray), and the derived Voronoi polygons are shown in orange.



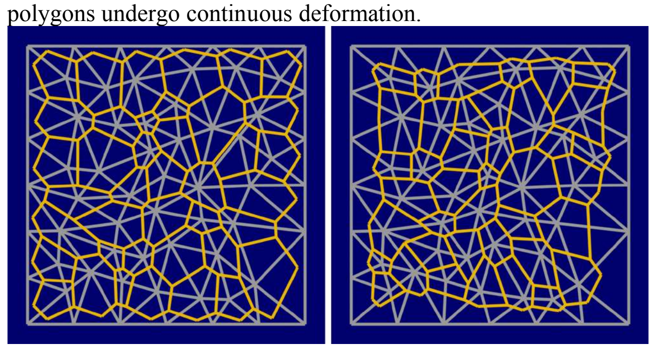
Note that Voronoi diagrams have been generalized in several ways - by using non-Euclidean distance metrics, associating weights with seeds to obtain curved edges, calculating distances from farthest points instead of the closest, using lines/shapes instead of seed points, etc. The technique described in the next section results in an entirely different set of variations, based on triangle centers of the Delaunay network. In fact, these variations could be combined with existing extensions to produce an even more rich variety of Voronoi-like alternatives.

Voronoi variations

In addition to the circumcenter, there are several other well-known triangle centers such as the centroid, incenter, orthocenter, etc. [Weisstein, E.]. **Any of these other triangle centers can be used on a set of Delaunay triangles, to obtain alternatives to the Voronoi polygon network.** As with the Voronoi network, these non-Voronoi polygons also share vertices and edges and for the most part, do not overlap or intersect - so they also create a polygon tiling. Voronoi polygons are always convex by definition, but these alternative polygons at times turn out to be concave. Also, judicious choices of extra inputs are necessary to prevent foldovers/overlaps (for those triangle centers that require additional parameters). In the following figure, the two Voronoi-like meshes result from computing incenters (left) and centroids (right). The difference between them is rather small (the barycenter version looks slightly less angular), but they differ significantly from the actual Voronoi diagram shown above.



The centroid of a triangle is the point where the triangle's three barycentric coordinates (vertex weights) are all equal to 1/3. This observation leads to the creation of 'deformed' versions of the centroid-based diagram, where the Voronoi-like mesh vertices are derived from arbitrary triplets of barycenter weights associated with the Delaunay seed points. A pair of such diagrams are shown below. On the left, for each triangle, the weights are (0.15,0.3,0.55) and on the right, they are (0.55,0.15,0.3). The weights can be smoothly varied in small steps, and the polygon network created at each step, to obtain an animation where the polygons undergo continuous deformation.



The tongue-in-cheek title of this sketch stems from the fact that in recent years, many more triangle centers have been discovered and cataloged on the Web. A collection of 2001 centers are presented in [Brisse], and an ever-growing list is maintained at [Kimberling] where the latest count is 3236 (some of these triangle centers have names such as Schiffler Point, Congruent Isoscelizers Point, Spieker Center, Nine-point Center and Gossard Perspector). The majority of these centers lie outside their triangles. But this is not a limitation for our network constructions since Voronoi circumcenters are not always contained within triangles either.

Three possible uses of these Voronoi alternatives are as follows. The centroid-based version could result in more natural-looking polygons for the purposes of brittle-fracturing a network along polygon edges. Conversely, a triangulation corresponding to a centroidal-Voronoi diagram (where each seed point coincides with the centroid of its enclosing Voronoi polygon, thereby creating evenly spaced/sized polygons) that uses incenters would be useful in creating artistic mosaic patterns and tilings. Local contraction and expansion of texture over a surface mesh can be obtained by covering it with a barycenter-based network and animating the barycentric weights, causing the mesh to deform.

References

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