Lecture 4

Space (axis) transformations

What have we seen so far?

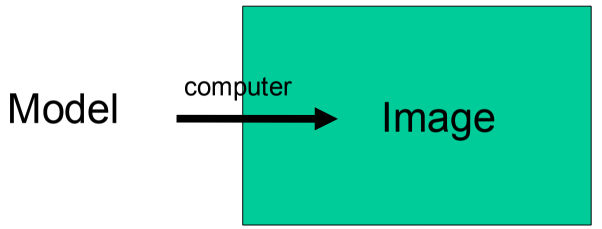
- Basic representations (point, vector)
- Basic operations on points and vectors (dot product, cross products, etc.)
- Transformation – manipulative operators on the basic representation (translate, rotate, deformations) – 4x4 matrices to "encode" all these.

Why do we need this?

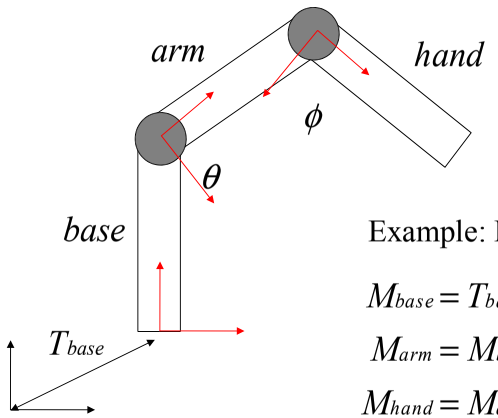
- In order to generate a picture from a model, we need to be able to not only specify a model (representation) but also manipulate the model in order to create more interesting images.

Overview

- The next set of slides will deal with the other half of the process (at least in a simplistic fashion)
- From a model, how do we generate an image



Accumulation of Transformations



Example: Robot arm

$$M_{base} = T_{base}$$

$$M_{arm} = M_{base}\, R(\theta) = T_{base}\, R(\theta)$$

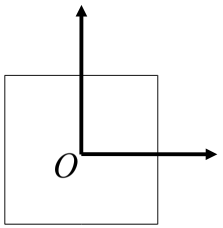
$$M_{hand} = M_{arm}\, R(\phi) = T_{base}\, R(\theta)\, R(\phi)$$

Coordinate Systems

- Object coordinates
- World coordinates
- Camera coordinates
- Normalized device coordinates
- Window coordinates

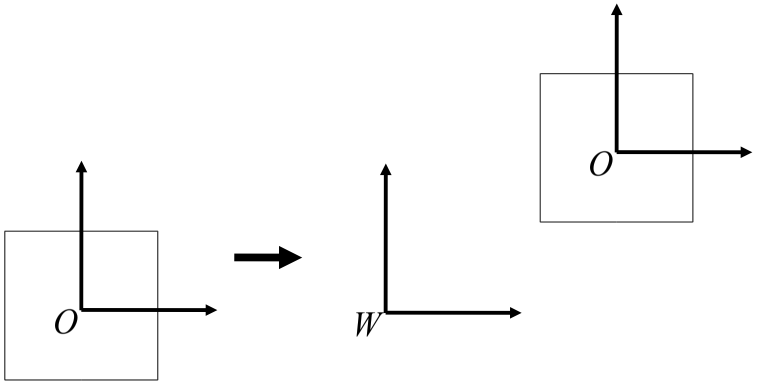
Object Coordinates

Convenient place to model the object



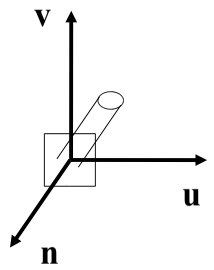
World Coordinates

Common coordinates for the scene



Camera Coordinates

Coordinate system with the camera in a convenient pose



$$M = \begin{bmatrix} u_x & u_y & u_z & -\mathbf{r}\cdot\mathbf{u} \\ v_x & v_y & v_z & -\mathbf{r}\cdot\mathbf{v} \\ n_x & n_y & n_z & -\mathbf{r}\cdot\mathbf{n} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Normalized Device Coordinates

Device independent coordinates

Visible coordinate usually range from:

$$-1 \le x \le 1$$
$$-1 \le y \le 1$$
$$-1 \le z \le 1$$

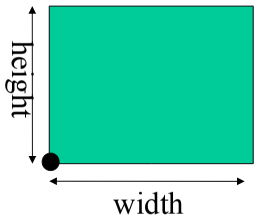
Window Coordinates

Adjusting the NDC to fit the window

(x_0, y_0) is the lower left of the window

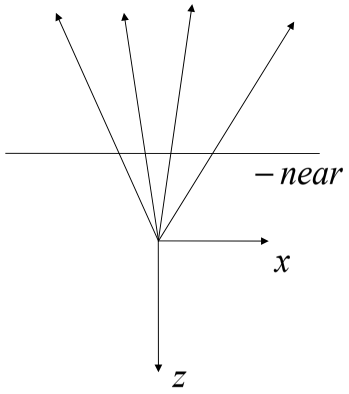
$$x_w = (x_{nd} + 1)\left(\frac{width}{2}\right) + x_0$$

$$y_w = (y_{nd} + 1)\left(\frac{height}{2}\right) + y_0$$

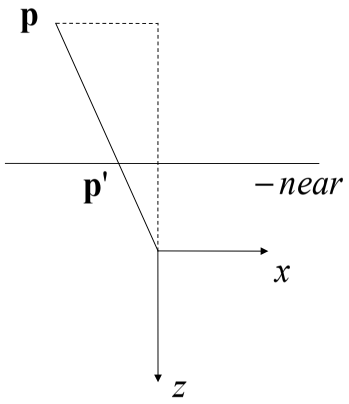


Perspective Projection

Taking the camera coordinates to NDC



Perspective Projection



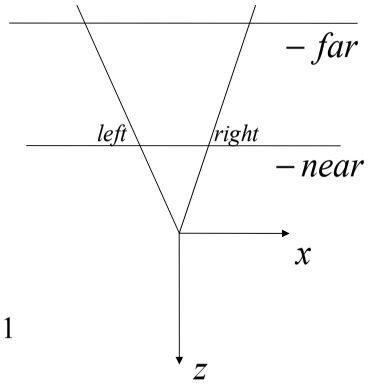
$$\frac{x'}{-near} = \frac{x}{z}$$

$$x' = -near\frac{x}{z}$$

Perspective Projection

Map (left,right) to (-1,1) when z = -near

$$x' = -near\frac{x}{z}$$



$$-near\frac{x}{z} - left$$

$$\frac{2}{right-left}\left(-near\frac{x}{z} - left\right)$$

$$\frac{2}{right-left}\left(-near\frac{x}{z} - left\right) - 1$$

$$\Rightarrow \quad \frac{-2near}{right-left}\frac{x}{z} - \frac{right+left}{right-left}$$

Pseudodepth

Map (-near,-far) to (-1,1)

$$z' = \frac{far+near}{far-near} + \frac{2\,far\cdot near}{far-near}\frac{1}{z}$$

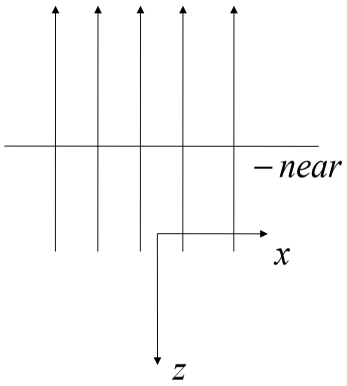
Lines are preserved through the transformation

See Newman and Sproull ’81 for the full derivation

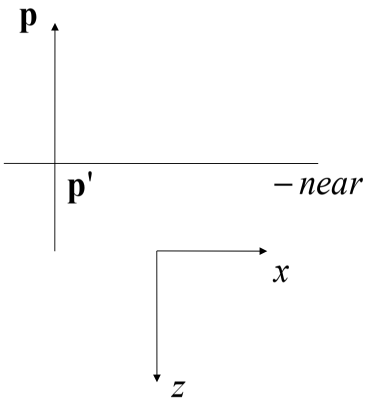
Perspective Projection

$$P = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\,far\cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Orthographic Projection



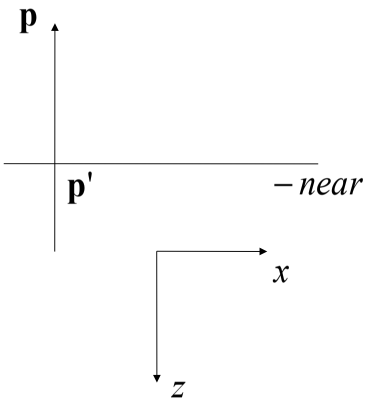
Orthographic Projection



$$x' = x$$

Orthographic Projection

Map (left,right) to (-1,1)



$$x' = x$$

$$x - left$$

$$\frac{2}{right - left}(x - left)$$

$$\frac{2}{right - left}(x - left) - 1$$

$$\frac{2}{right - left}x - \frac{right + left}{right - left}$$

Orthographic Projection

Map (near,far) to (-1,1)

$$z$$

$$z - near$$

$$\frac{2}{far - near}(z - near)$$

$$\frac{2}{far - near}(z - near) - 1$$

$$\frac{2}{far - near}z - \frac{far + near}{far - near}$$

Orthographic Projection

$$P = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic Projections Preserve Straight Lines

$$\begin{bmatrix} a_x + b_x t \\ a_y + b_y t \\ a_z + b_z t \\ 1 \end{bmatrix} \text{ maps to } \begin{bmatrix} A(a_x + b_x t) + B \\ C(a_y + b_y t) + D \\ E(a_z + b_z t) + F \\ 1 \end{bmatrix}$$

Putting it all together!!

- Take your representation (points) and transform it from Object Space to World Space
- Take your World Space point and transform it to Camera Space
- Perform the remapping and projection onto the image plane in Normalized Device Coordinates
- Perform this set of transformations on each point of the polygonal object
- "Connect the dots" through line rasterization

Intuitively